MAT 121 §3.4 #s 6, 11, 16, 18, 22, 37, 38, 44, 45

#s 3-40 Solve each inequality

(6) $x^3 + 8x^2 < 0$

$x^3 + 8x^2 = x^2(x+8)$

$x = 0$ $m = 2$ Touch

$x = -8$ $m = 1$ Cross

We want the <0 part:

$\boxed{(-\infty, -8)}$

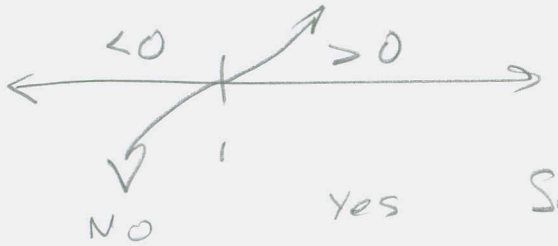
(11) $(x-1)(x^2+x+4) \geq 0$

Check the x^2+x+4 piece (Descartes will tell us, later, that we won't have any real zeros)

$a = 1, b = 1, c = 4 \Longrightarrow$

$b^2 - 4ac = 1^2 - 4(1)(4) = -15 < 0$

No real zeros.



E.B.: x^3 ... ↗

So, $\boxed{x \in [1, \infty)}$ satisfies $(x-1)(x^2+x+4) \geq 0$

(16) $x^3 + 2x^2 - 3x > 0$

$\Longrightarrow x(x^2 + 2x - 3) > 0$

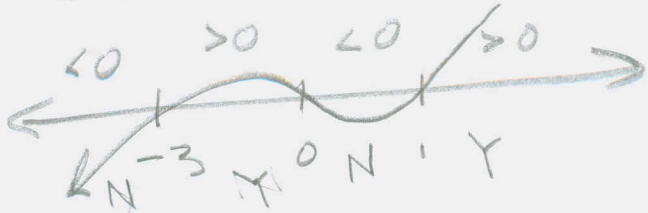
$\Longrightarrow x(x+3)(x-1) > 0$

$x = -3$ $m = 1$

$x = 0$ $m = 1$ } Cross

$x = 1$ $m = 1$

E.B.: x^3



... ↗

MAT 121 § 3.4 #s 16, 18, 22, 37, 38, 44, 45

(16)

-3 No, 0 Yes, 1 No, Yes

$(-\infty,-3), (-3,0), (0,1), (1,\infty)$

Want "+" for ">0": $\boxed{x \in (-3,0) \cup (1,\infty)}$

satisfies $x^3+2x^2-3x>0$

(18) $x^4 < 9x^2$

$\Rightarrow x^4 - 9x^2 < 0$

$\Rightarrow x^2(x^2-9) < 0$

$\Rightarrow x^2(x+3)(x-3) < 0$

$x=-3$ $m=1$ cross

$x=0$ $m=2$ touch

$x=3$ $m=1$ cross

E.B.: x^4

-3 0

No Yes Yes No

Want "<0"

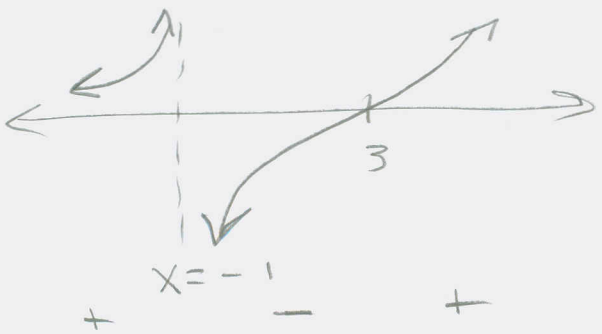
Want "$-$"

So, $\boxed{x \in (-3,0) \cup (0,1)}$ Satisfies $x^4 < 9x^2$

(22) $\frac{x-3}{x+1} > 0$

$x=3$ $m=1$ } Cross

$x=-1$ $m=1$ }



Want ">0"

Want "$+$"

So, $\boxed{x \in (-\infty,-1) \cup (3,\infty)}$

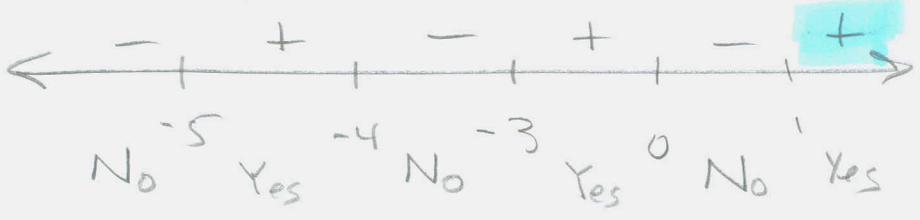
Satisfies $\frac{x-3}{x+1} > 0$

MAT 121 §3.5 #s 37, 38, 44, 45

(37) $\dfrac{x^2(3+x)(x+4)}{(x+5)(x-1)} \geq 0$

is EXACTLY like $x^2(3+x)(x+4)(x+5)(x-1) \geq 0$

EXCEPT we can't let $x=-5$ OR $x=+1$, because of the domain.



$x=-5$ cross
$x=-4$ cross
$x=-3$ cross
$x=0$ TOUCH
$x=1$ cross

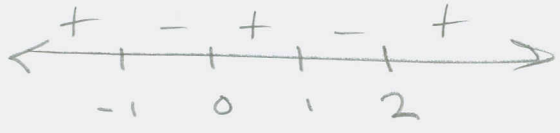
So $\boxed{x \in (-5,-4] \cup [-3,0] \cup (1,\infty)}$

(Note; $x=-5$ & $x=1$ must be excluded!)

$f(2) = \dfrac{2^2(3+2)(2+4)}{(2+5)(2-1)} = \dfrac{2^2(5)(6)}{(7)(1)}$ is "+"

→ $x \in \{x \mid -5 < x \leq -4 \text{ OR } -3 \leq x \leq 0 \text{ OR } 1 < x\}$ is another version of same answer.

(38) $\dfrac{x(x^2+1)(x-2)}{(x-1)(x+1)} \geq 0$



Can't let $x=1$ OR $x=-1$

$\boxed{x \in (-\infty,-1) \cup [0,1) \cup [2,\infty)}$

MAT 121 S' 3.4 #s 44, 45

(44) What's the domain of $\sqrt{x^3 - 3x^2}$?

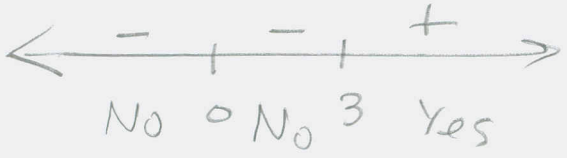
$\mathcal{D} = \{x \mid x^3 - 3x^2 \geq 0\}$.

We solve $x^3 - 3x^2 \geq 0$

$x^2(x-3) \geq 0$

$x=0$ $m=2$ touch

$x=3$ $m=1$ cross

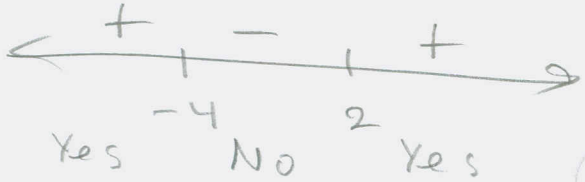


So, $\boxed{x \in [3, \infty) = \mathcal{D}}$

(45) What's the domain of $f(x) = \sqrt{\frac{x-2}{x+4}}$?

Need $\frac{x-2}{x+4} \geq 0$ AND can't let $x+4=0$

$\Rightarrow x = -4$ is BAD



So $\mathcal{D} = (-\infty, -4) \cup [2, \infty)$

That's the reason for the parentheses instead of the square bracket.